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Hybrid choice models: Principles and recent progress incorporating social influence and nonlinear utility functions

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Abstract

Hybrid choice models have been developed as an extension of discrete choice models, particularly multinomial logit models, in an attempt to include attitudinal variables. The quintessence of hybrid choice models is that a model of attitude formation is estimated and the estimated attitudes are added to the commonly used set of attributes in discrete choice models: attributes of the choice alternatives and socio-demographic variables. The most commonly applied model is based on linear specifications, both for the attitude model and the utility function. In this review paper, we discuss the principles underlying the hybrid choice model, summarize the specifications used in previous applications of the model and then continue discussing recent progress that added social influence to the model specification and replaced the linear specification of the utility function with a nonlinear function.

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Keywords: Hybrid choice model, social influence, nonlinear functions.

1. Introduction

Discrete choice analysis has become a standard approach for the analysis of activity-travel choice behaviours, such as travel mode, residential and activity locations, car ownership, and so forth^{1,2}. Numerous authors have proposed alternative discrete choice models to represent a behaviourally more realistic choice process. However, it is not easy to completely understand and represent a decision-maker's choice behaviour because a considerable number of difficult to measure factors, such as decision-makers' latent attitudes, tastes, perceptions, beliefs, values, etc. may influence the choice process. These unobservable factors and the causal relationships among them are

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difficult to identify. The hybrid choice model (HCM) represents an attempt to identify such unobservable factors and include them into a discrete choice analysis.

In the present paper, we discuss the principles underlying the HCM. The paper serves two aims. First, we give an overview of the HCM framework, which should be helpful for readers new to the field in understanding the original HCM formulation. Second, we review several variations of the HCM, including the authors' elaborations of its sub-frameworks. These efforts allow different representations of choice behaviour. In particular, first, we focus on the model incorporating social influences. Social influence can be defined as the influence of choices of social network members. Second, we show how the model can accommodate a nonlinear utility function specification. Assuming nonlinear relationships is theoretically more appealing because it represents a more general approach to model specification.

This paper is organized as follows. The following section presents an overview of the original framework, underlying the HCM, and commonly used formulations and estimation methods. In the third section, recent progress in accommodating social influence and nonlinear utility function are explained. The last section presents a discussion and conclusion.

2. Overview of the hybrid choice model

HCM can be viewed as an expanded discrete choice modeling framework, which integrates different types of models into a single structure that is estimated simultaneously³. Basically, HCMs incorporate a latent variable model into a discrete choice model in order to improve the explanatory power of the choice model by considering the effects of decision makers' latent attitudes. The HCM framework is illustrated in Fig. 1. The ellipses represent unobservable variables, while the rectangles represent observable variables. Each of these sub-models comprises a structural component and a measurement component. Since the latent attitudes (i.e. latent variables) cannot be directly observed from revealed choices, they should be identified through a set of attitudinal indicators. The latent variable model permits identifying latent constructs as a function of the indicators, and capture the causal relationships between exogenous explanatory variables and the latent variables. By simultaneously integrating discrete choice and latent variable models, the latent variables can be treated as explanatory variables in the utility functions of choice alternatives. According to the models included, this structure has also been referred to as the integrated choice and latent variable (ICLV) model⁴.

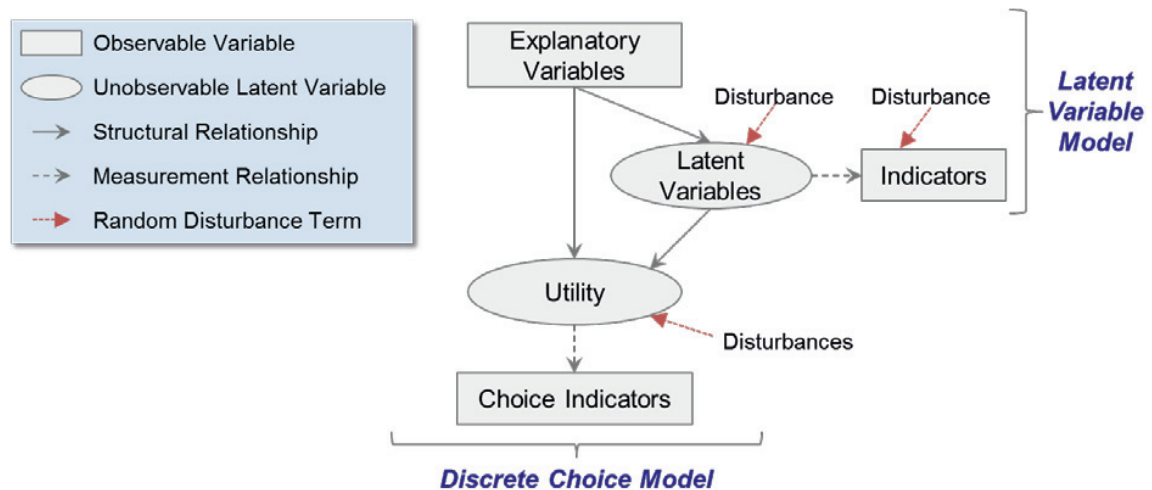


Fig. 1. Framework for hybrid choice model: an integrated discrete choice and latent variable model (adapted from Ben-Akiva et al.⁵).

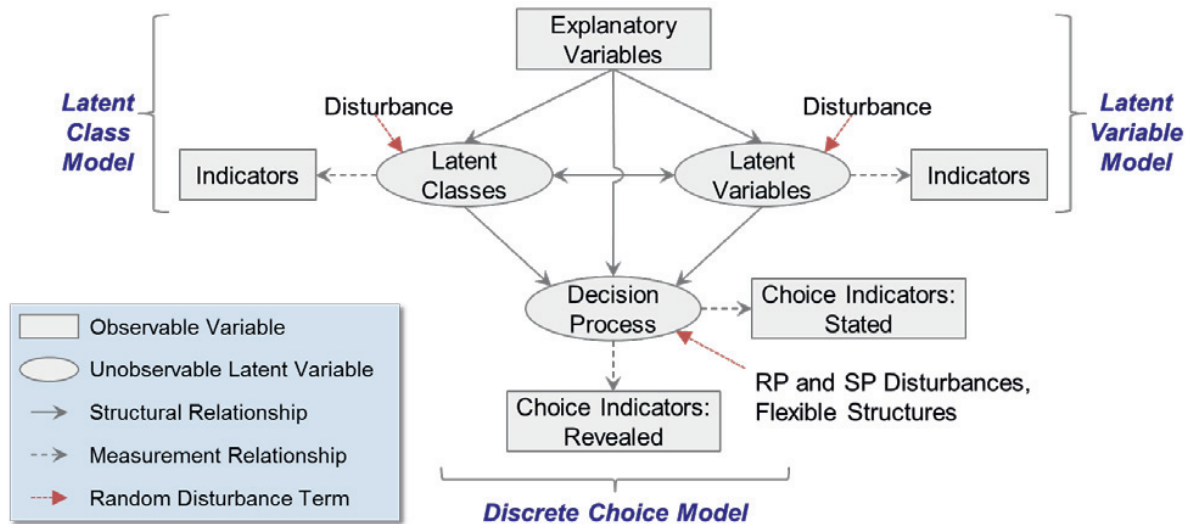


Fig. 2. Framework for hybrid choice model: an expanded model integrating choice, latent variable and latent class models with flexible error structures (adapted from Ben-Akiva et al.³).

Earlier efforts to incorporate latent variables into discrete choice models included error-free attitudinal indicators directly in the utility functions⁶, using a factor analysis of the indicators⁷, and employing latent attributes without the indicators⁸. These early approaches, however, have some limitations. Including the indicators directly in the utility function can lead to inconsistent estimates because this approach ignores the fact that latent variables contain measurement error. In addition, there may be endogeneity bias caused by correlation between the indicators and the error of the utility. Unobserved factors can influence a respondent not only to choose an alternative but also to respond to indicator questions. Using a factor analysis can be an alternative approach to overcome these limitations, which is a sequential approach: a factor analysis is carried out to identify latent variables, and then the latent variables are included in the utility function. However, the estimated latent variables can be inefficient because the choice indicators (i.e. the actual choice behaviors of respondents) are not considered when estimating the latent variables. In the approach employing latent attributes without the indicators, choice indicators only measure both the latent attributes and the utility. Thus, the latent attributes are treated as alternative-specific and do not vary among individuals in a market segment. Ben-Akiva et al.⁵ and Daly et al.⁹ provide a review of these early approaches.

Alternatively, structural equation models (SEMs) have been used for simultaneously considering latent attitudes and choice behaviour. Golob¹⁰ gives a review of these models; Sakano and Benjamin¹¹ describe an application. The SEM can simultaneously estimate the causal influence of exogenous variables on endogenous latent variables and the causal relationships among endogenous latent variables. At the same time, the latent variables can be identified through observed indicators. Thus, SEMs allow capturing the effects of latent attitudes on choice behaviour by treating the latent attitudes and the utilities of choice alternatives as endogenous latent variables and by employing choice indicators to identify the utilities. Although this framework is very similar to HCM, the SEMs have some limitations. Typically, SEMs have been developed based on linear-in-parameters multivariate statistical modeling techniques (i.e., linear regression techniques), mostly for continuous latent variables and continuous indicators. Therefore, when including discrete indicators, SEMs result in inconsistent estimates. Although some corrective procedures to allow various types of indicators have been suggested¹², these approaches are limited to binary choice and ordered discrete indicators.

The HCM framework is methodologically more satisfactory than SEM because more diverse representations of choice behaviour can be incorporated into a mathematical model. HCMs can be expanded to deal with heterogeneities due to latent market segmentation and individual taste variation by introducing a latent class model and mixed logit model respectively¹³.

Table 1. Overview of previous researches employing the HCMs to investigate travel choice behaviours

Authors	Application Subject	Type of preference ¹⁾ : Choice set	Latent variables	Types of sub-frameworks ²⁾		Estimation approach ³⁾
				Discrete choice	Latent variable	
Walker and Ben-Akiva ¹³	Travel Mode	RP: (1) rail, (2) auto SP: 5 scale ordered preference of rail	(1) comfort, (2) convenience	Mixed Logit (for TV & PE)	MIMIC	MSL
Temme et al. ¹⁷		RP: (1) public transit, (2) car + public transit, (3) public transit only	(1) flexibility, (2) convenience and comfort, (3) safety, (4) power, (5) hedonism, (6) security	Logit	MIMIC	MSL ⁴⁾
Kim et al. ¹⁸		SP: (1) auto, (2) train, (3) water transit	(1) environmental preferences, (2) dislike of driving a car, (3) water transit preference	Logit	MIMIC	ML: numerical integration
Atasoy et al. ¹⁹		RP: (1) private motorized modes, (2) public transport, (3) soft modes	(1) pro-car, (2) environmental concern	Logit	MIMIC	ML (not specified)
Kamrgianni et al. ²⁰		RP: (1) walk, (2) bus, (3) car	(1) walking-lover, (2) parents' walking-lover	Logit	MIMIC	MSL
Daziano and Bolduc ²¹	Vehicle (fuel) type	SP: (1) standard gas vehicle, (2) alternative fuel vehicle, (3) hybrid vehicle, (4) hydrogen fuel cell vehicle	(1) environmental concerns	Logit	MIMIC	Bayesian approach
Glerum et al. ²²		SP: (1) Competitors-gasoline, (2) Renault-gasoline, (3) Renault-electric	(1) pro-leasing attitude, (2) pro-convenience attitude	Logit	MIMIC	ML: numerical integration
Jensen et al. ²³		SP: (1) conventional, (2) electric vehicle	(1) environmental attitude	Mixed Logit (for PE)	MIMIC	MSL
Mabit et al. ²⁴		SP: (1) conventional vehicle, (2) hybrid non-plugin vehicle, (3) bio-diesel vehicle, (4) electric vehicle	(1) appreciation of car features	Logit	MIMIC	ML: numerical integration
Soto et al. ²⁵		SP: (1) standard gasoline vehicle, (2) natural gas vehicle, (3) hybrid electric vehicle, (4) electric vehicle or diesel vehicle	(1) support transport policies, (2) environmental concern, (3) attitudes towards technology, (4) attitudes towards car	Mixed Logit (for PE)	Ordered Logit	MSL
Prato et al. ²⁶	Travel route	RP: 2 ~ 19 alternative routes per observation (generated by the branch and bound algorithm)	(1) mnemonic ability, (2) habit within the choice environment, (3) familiarity with the choice environment, (4) spatial ability, (5) time saving skill	Logit	MIMIC	MSL
Daly et al. ⁹		SP: 4 options including "not to choose"	(1) increased concern, (2) reduced distrust	Mixed Logit (for PE)	Ordered Probit	MSL
Silva et al. ²⁷	Tour type	RP: (1) home-work/school-home (HWH), (2) HWH + other activities, (3) home-other-home (HOH), (4) HOH + other activities	(1) travel propensity	Logit	MIMIC	ML: numerical integration
Dekker et al. ¹⁴	Leisure activity	SP: (1) museum visit, (2) concert visit, (3) nature walk	(1)-(3) the needs satisfaction of museum visit, concert visit, and nature walk	Hybrid RUM-RRM (Mixed Logit for PE)	Ordered Probit	Bayesian approach

Note 1) RP: revealed preference, SP: stated preference

2) TV: individual taste variation, PE: panel effect, MIMIC: multiple indicators multiple causes model

3) ML: maximum likelihood approach, MSL: maximum simulated likelihood approach

4) They did not mention MSL but ML based on Monte Carlo integration. The approach used can be viewed as a simulation-based estimation approach like MSL.

The expanded HCM framework is illustrated in Fig. 2. It includes a sub-framework to combine two types of preference data: revealed preferences (RP), which are based on actual market behaviour, and stated preferences (SP), which are based on hypothetical choice situations. Furthermore, in terms of the assumed decision process, recently, several researchers¹⁴ suggested replacing the random utility maximization model (RUM) with the random regret minimization model (RRM)¹⁵ or with a hybrid RUM-RRM model¹⁶.

Over the last decade, numerous studies have developed HCMs to investigate the effects of people's latent attitudes on diverse travel choice behaviours. Table 1 provides a summary of previous research. It can be classified according to their applications, model specification (i.e. types of sub-frameworks), and estimation approaches.

2.1. Applications and their latent variables

In terms of travel mode choice behaviour, Walker and Ben-Akiva¹³ considered subjective perceptions of comfort and convenience for different transportation modes as latent attitudes. Temme et al.¹⁷ considered perceptions with respect to flexibility, convenience, comfort and safety and the personal values (i.e. power, hedonism and security). Kim et al.¹⁸ and Atasoy et al.¹⁹ employed the personal attitudes toward the environment (i.e. environmental preferences or concerns) as latent attitudes, which may influence the utility of the transportation model. In addition, they considered people's latent attitudes toward specific travel modes such as pro-car attitude, dislike of driving a car and water transit preference. Kamargianni et al.²⁰ considered attitudes toward walking. Moreover, they included parents' latent preference for walking, which may influence the walking preferences of their children.

In terms of vehicle (fuel) type choice, Daziano and Bolduc²¹ and Jensen et al.²³ employed the attitude toward the environment as a crucial latent variable, representing the utilities of alternative fuel vehicles (e.g., electric vehicle, hybrid vehicle, hydrogen fuel cell vehicle, etc.). Glerum et al.²² considered pro-leasing attitude and pro-convenience attitude (e.g., spaciousness, technology, etc.) when modeling purchase behaviour of electric car. Mabit et al.²⁴ included personal appreciation of car features (e.g., joy of driving, comfort, design, etc.) as latent variables in their HCM. Soto et al.²⁵ used more diverse latent attitudes including personal attitudes toward transport policies, environmental concerns, and attitudes towards technology and car.

Prato et al.²⁶ studied travel route choice behaviour by developing a HCM, which included 5 different personal psychological abilities and characteristics as latent variables. Daly et al.⁹ employed personal distrust of government as a latent variable for investigating people's stated preference for different rail security systems. Silva et al.²⁷ investigated the effects of personal propensity to travel on tour type choice behaviour. The result shows that the propensity increases the probability to perform more complex tours. Dekker et al.¹⁴ addressed leisure activity choice behaviour. They employed the needs satisfaction of each leisure activity as latent variables in the discrete choice analysis. Results of these applications of HCMs provide evidence of the additional insight in decision-making processes HCM models can provide and their explanatory power.

2.2. Model specification: the basic model

A basic HCM incorporates a latent variable model specified by linear-in-parameter relationships into a discrete choice model based on linear utility functions. Most previous research has employed the binary/multinomial logit model and the multiple indicators multiple causes (MIMIC) model as the discrete choice model and the latent variable model, respectively. The MIMIC model is a set of simultaneous equations based on linear-in-parameter specifications, in which a latent variable is measured by multiple indicators and regressed on several observable exogenous variables. The model consists of a set of structural and measurement relationships. The structural relationship indicates the causal relationships between observable exogenous variables (e.g., decision-maker's socio-demographics) and latent variables. In the measurement relationship, multiple indicator variables are used to identify the latent variables. The indicators are responses to survey questions regarding different attitudes. The structural equation (1) and the measurement equation (2) can be expressed as follows:

$$X_n^L = \Gamma^{LZ} X_n^Z + \zeta_n^L, \quad \zeta_n^L \sim N(0, \sigma_{\zeta^{LL}}) \quad (1)$$

$$\mathbf{I}_n^D = \mathbf{\Lambda}^{DL} \mathbf{X}_n^L + \boldsymbol{\varepsilon}_n^D, \quad \boldsymbol{\varepsilon}_n^D \sim N(0, \boldsymbol{\sigma}_{\varepsilon^{DD}}) \quad (2)$$

where \mathbf{X}_n^L indicates an $(L \times 1)$ vector of L latent variables of individual n , and \mathbf{X}_n^Z is a $(Z \times 1)$ vector of Z observable explanatory variables. The $(L \times Z)$ matrix $\mathbf{\Gamma}^{LZ}$ contains unknown parameters, and the random part $\boldsymbol{\zeta}_n^L$ indicates a multivariate normal distribution with zero mean and an $(L \times L)$ covariance matrix $\boldsymbol{\sigma}_{\zeta^{LL}}$. In order to identify latent variables, a $(D \times 1)$ vector of observable indicator variables \mathbf{I}_n^D is utilized, which are responses of individual n to D survey questions regarding different latent attitudes. This measurement relationship can be expressed by a $(D \times L)$ matrix of unknown parameters $\mathbf{\Lambda}^{DL}$ and random disturbance term $\boldsymbol{\varepsilon}_n^D$ which is also a multivariate normal distribution with zero mean and an $(D \times D)$ covariance matrix $\boldsymbol{\sigma}_{\varepsilon^{DD}}$.

The logit model is also comprised of a structural relationship and a measurement relationship. The structural relationship is manifested in the utility functions of the choice alternatives, which consists of the latent variables and the observed exogenous variables. In previous research, the structural relationships have been represented as linear-in-parameters utility functions. The measurement relationship is derived under the assumption of utility-maximizing behaviour. In order to express choice as a function of utility, an individual's observed choice is used as a single nominal indicator. Structural equation (3) and measurement equation (4) can be expressed as follows:

$$U_{in} = \boldsymbol{\beta}_Z(i) \mathbf{X}_n^Z(i) + \boldsymbol{\beta}_L(i) \mathbf{X}_n^L(i) + \boldsymbol{\beta}_M(i) \mathbf{X}_n^M(i) + \varepsilon_{in}, \quad \varepsilon_{in} \sim G(0, \sigma_{\varepsilon_i}) \quad (3)$$

$$y_{in} = \begin{cases} 1, & \text{if } U_{in} = \max_j \{U_{jn}\} \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where U_{in} is the random utility of alternative i of individual n . The notation (i) after a variable matrix indicates the i th column vector corresponding to the variable matrix. Notation (i) after a parameter matrix indicates the i th row vector corresponding parameter matrix. Thus, the notation (i) represents the components in terms of the i th alternative in each variable and parameter matrices. In the discrete choice model, the dimensions of \mathbf{X}_n^Z and \mathbf{X}_n^L are expanded to explain the utility corresponding to each alternative for individual n ; thus, the dimensions become $(Z \times J)$ and $(L \times J)$. A $(M \times J)$ matrix \mathbf{X}_n^M indicates alternative-attribute variables of each alternative of individual n . The $(J \times Z)$ matrix $\boldsymbol{\beta}_Z$, the $(J \times L)$ matrix $\boldsymbol{\beta}_L$, and the $(J \times M)$ matrix $\boldsymbol{\beta}_M$ are unknown parameters to be estimated for socio-demographic, latent-attitudes and alternative-attributes, respectively. ε_{in} represents the random disturbance term with zero mean and standard deviation σ_{ε_i} . The logit model is induced by assuming independently, identically (IID) standard Gumbel distributed random disturbances.

The joint likelihood function of the basic HCM can be defined by assuming that the random disturbance terms are independent. The likelihood of an individual consists of the likelihood function of the discrete choice model (i.e. logit model) and the measurement and structural components of the latent variable model (i.e. MIMIC model) as follows:

$$\mathbb{L}_n(\mathbf{y}_n, \mathbf{I}_n^D | \mathbf{X}_n^Z, \mathbf{X}_n^L, \mathbf{X}_n^M; \boldsymbol{\theta}) = \int_L f_y(\mathbf{y}_n | \mathbf{X}_n^Z, \mathbf{X}_n^L, \mathbf{X}_n^M; \boldsymbol{\beta}_Z, \boldsymbol{\beta}_L, \boldsymbol{\beta}_M, \boldsymbol{\sigma}_{\varepsilon}) f_I(\mathbf{I}_n^D | \mathbf{X}_n^L; \mathbf{\Lambda}^{DL}, \boldsymbol{\sigma}_{\varepsilon^{DD}}) f_L(\mathbf{X}_n^L | \mathbf{X}_n^Z; \mathbf{\Gamma}^{LZ}, \boldsymbol{\sigma}_{\zeta^{LL}}) d\mathbf{L} \quad (5)$$

where $\boldsymbol{\theta} = \{\boldsymbol{\beta}_Z, \boldsymbol{\beta}_L, \boldsymbol{\beta}_M, \mathbf{\Lambda}^{DL}, \mathbf{\Gamma}^{LZ}, \boldsymbol{\sigma}_{\varepsilon}, \boldsymbol{\sigma}_{\varepsilon^{DD}}, \boldsymbol{\sigma}_{\zeta^{LL}}\}$ is the full set of parameters to estimate. $f_y(\cdot)$ indicates the likelihood function of the discrete choice model, $f_I(\cdot)$ and $f_L(\cdot)$ indicate the distribution functions with regard to the latent variable model, which correspond to the measurement relationship and the structural relationship, respectively. Since the latent variables are unknown, the likelihood can be obtained by integrating over the latent constructs $f_L(\cdot)$. Thus, the dimension of the integral becomes the same as the number of latent variables. By jointly constructing the indicators in the latent variable model with the distribution function of the measurement relationship, the indicators do not only allow identifying the latent variables, but also provide efficiency in estimating the full model. The functional forms of these sub-functions depend on the assumptions about the probability distributions of their random disturbance terms. According to the assumptions of the probability distributions of the disturbance terms (i.e. equations (1), (2) and (3)), the sub-functions can be induced as the following equations:

$$f_y(y_n | X_n^Z, X_n^L, X_n^M, \beta_Z, \beta_L, \beta_M, \sigma_\varepsilon) = \prod_{i \in J} \left(\frac{\exp(\beta_Z(i)X_n^Z(i) + \beta_L(i)X_n^L(i) + \beta_M(i)X_n^M(i))}{\sum_{j \in J} \exp(\beta_Z(j)X_n^Z(j) + \beta_L(j)X_n^L(j) + \beta_M(j)X_n^M(j))} \right)^{y_{in}} \quad (6)$$

$$f_I(I_n^D | X_n^L; \Lambda^{DL}, \sigma_{\varepsilon^{DD}}) = \prod_{d \in D} \frac{1}{\sigma_{\varepsilon^d}} \Phi \left(\frac{I_n^d - \Lambda(d)^{DL} X_n^L}{\sigma_{\varepsilon^d}} \right) \quad (7)$$

$$f_L(X_n^L | X_n^Z; \Gamma^{LZ}, \sigma_{\zeta^{LL}}) = \prod_{l \in L} \frac{1}{\sigma_{\zeta^l}} \Phi \left(\frac{X_n^l - \Gamma(l)^{LZ} X_n^Z}{\sigma_{\zeta^l}} \right) \quad (8)$$

where $\Phi(\cdot)$ indicates the standard normal density function. While the covariance matrix σ_ε needs not to be estimated according to the normalization with IID error for the logit model, the covariance matrices $\sigma_{\varepsilon^{DD}}$ and $\sigma_{\zeta^{LL}}$ have to be estimated. If the disturbance terms are assumed to be IID, the covariance matrices are diagonal matrices whose off-diagonal elements are equal to zero. Accordingly, only σ_{ε^d} and σ_{ζ^l} should be estimated, which are the standard deviations of the disturbance terms indicating d th and l th diagonal elements of $\sigma_{\varepsilon^{DD}}$ and $\sigma_{\zeta^{LL}}$, respectively. $\Lambda(d)^{DL}$ indicates the d th row vector of the $(D \times L)$ unknown parameter matrix Λ^{DL} , and $\Gamma(l)^{LZ}$ indicates the l th row vector of the $(L \times Z)$ unknown parameter matrix Γ^{LZ} .

2.3. Model specification: mixed logit specification as the discrete choice model

The mixed logit model is a flexible model, which can approximate any random utility model by considering the probability density of parameters²⁸. The model allows considering individual taste variation, unrestricted substitution patterns, correlation in unobserved factors over time or repeated choice situations (i.e. panel effect)²⁹. Walker and Ben-Akiva¹³ have included the mixed logit model in the HCM in order to reflect individual taste variations in terms of alternative attributes and the panel effect due to the repeated choices of each individual in a stated choice experiment. Random coefficients for the parameters of the attributes can represent individual taste variations. The random coefficients vary across individuals according to their assumed probability density functions. Similarly, the panel effect can be represented by a random coefficient, which vary across individuals but is constant over choice situations for each individual. Daly et al.⁹, Jensen et al.²³, Soto et al.²⁵ and Dekker et al.¹⁴ also treated the panel effect by introducing the mixed logit specification into the HCM. The HCM allowing the mixed logit specification has more explanatory power than the basic HCM.

2.4. Model specification: ordered choice model as the latent variable model

The attitudinal indicators are used to identify people's latent attitudes. Normally, they are responses to attitudinal questions and psychological dispositions included in surveys. For example, in terms of environmental attitudes, questions such as "I'm likely to choose an alternative mode in order to protect the environment" and/or "I'm willing to pay more for environmentally friendly items" can be utilized¹⁸. Based on these attitudinal questions, a respondent judges the degree of agreement of each statement. Likert scales are most widely used to measure attitudes (e.g., from "strongly disagree" to "strongly agree").

The basic HCM, which includes a MIMIC model as the latent variable model, assumes continuous indicators and linear relationships between the latent attitudes and their indicators. However, Daly et al.⁹ have pointed out that these assumptions are not consistent with the ordinal nature of the measurement of the indicators. They therefore suggested replacing the continuous specification by an ordered specification. By treating the indicators as ordered categorical variables, the latent variable model part can be represented by an ordered choice model. They concluded that this specification is superior to the general assumption of a continuous attitudinal response because using the ordered model contributes to an improved explanation of choice behaviour and is intuitively more consistent with the nature of the response for the attitudinal indicator. Soto et al.²⁵ also considered the indicators as ordered categorical variables. They jointly estimated an ordered choice model to identify latent attitudes and a mixed logit

model to investigate choice behaviour in terms of alternative fuel vehicles. Dekker et al.¹⁴ employed ordinal indicators to measure personal satisfactions of each activity in terms of six potential types of need. In order to reflect the ordinal nature of the satisfaction, they included an ordered probit model as the latent variable model in the HCM framework.

2.5. Model specification: hybrid random utility maximization-random regret minimization model as the discrete choice model

The random regret minimization (RRM) model was developed based on the behavioral intuition that individuals try to avoid a situation where one or more non-chosen alternatives outperform a chosen one in terms of one or more attributes or characteristics¹⁵. The model postulates that people choose an alternative, which minimizes anticipated random regret. The most-used framework of the RRM model³⁰ takes the linear-additive multinomial logit form by replacing utility with the negative value of anticipated regret. The anticipated regret of an alternative is equal to the sum of the pairwise regrets which are obtained by comparing the considered alternative with each of the other alternatives in the choice set for every attribute. The RRM model can capture semi-compensatory choice behaviour and predict choice set composition effects such as the compromise effect³¹. There have been several empirical studies to investigate travel choice behavior using RRM models. For instance, Chorus³⁰ applied the RRM model for understanding travel mode and route choice, travel information acquisition, parking lot choice and shopping destination choice behaviours, Chorus and de Jong³² employed the RRM model to investigate people's departure time choice behaviour for their daily commute, Hensher et al.³³ studied vehicle type choice behaviour using the RRM model, and Chorus and Bierlaire³⁴ established the RRM model for modeling route choice behaviour.

In order to reflect the heterogeneity of decision making along the attributes into the RRM approach, Chorus et al.¹⁶ suggested a hybrid RUM-RRM model based on a random modified utility function which consists of the random utility and the random regret parts. This modified utility function allows for representing difference choice processes by difference attributes, which means that some attributes are processed based on the utility maximization rule, while others are processed based on the regret minimization rule. According to the empirical analyses of Chorus et al.¹⁶, the hybrid RUM-RRM models leads to statistically significant improvements in model fit compared with both the full utility-based model and the full regret-based model.

Dekker et al.¹⁴ employed a hybrid RUM-RRM model as the discrete choice model of the HCM. In their modified utility function, the needs satisfactions for each activity (i.e. latent variables) were treated as the attributes to be processed using the utility maximization rule, and the observable attributes (e.g., travel time and costs, activity costs, etc.) were treated as the attributes to be processed using the regret minimization rule. However, their empirical results show that this specification is not significantly better than the full utility-based specification in terms of model fit and predictive ability.

Although this dominant specification based on pairwise comparisons is easy to apply, it has some fundamental flaws when attribute differences are small³⁵. Moreover, Rasouli and Timmermans³⁶ did not find much support for the logarithmic transformation of the regret function nor for the comparison against all alternatives compared to the classic formulation of the regret model¹⁵ in which regret was defined in terms of a function of attribute differences against the best foregone choice alternative.

2.6. Estimation methods

Three different estimation methods have been to estimate HCM: the maximum likelihood (ML) based on numerical integration^{18,22,24,27}, simulated maximum likelihood (MSL)^{9,13,17,20,23,25,26}, and Bayesian estimation^{14,21}.

The ML based on numerical integration directly calculates the likelihood value by numerically integrating the joint likelihood function (e.g., equation (5)). However, because the dimension of the joint likelihood function is determined by the number of latent variables, this approach quickly becomes infeasible as the number of latent variables increases. In the case of more than three latent variables, a simulation-based estimation approach such as MSL and Bayesian estimation are more appropriate, because these methods are not sensitive to the number of latent variables. MSL is the same as ML, except that simulated probabilities rather than exact probabilities are used. The simulated probabilities are obtained by random draws of the latent variables from their probability distributions.

MSL takes the average value of the likelihood values corresponding to each random draw as an unbiased estimator. As shown in Table 1, the MSL method has been predominantly used in the previous research. Although it provides consistent and efficient estimates regardless of the number of latent variables, Daziano and Bolduc²¹ pointed out that the MSL method for HCM is very demanding in situations with a huge choice set and a large number of latent variables. In order to overcome these limitations, the simulated mean of the Bayesian posterior (SMP) can be employed as an alternative estimator to the MSL estimator. In the Bayesian approach, the SMP can be obtained through a Markov Chain Monte Carlo technique, which is an iterative sampling procedure that the random draws at each iteration depend on those at the preceding iteration (e.g., Gibbs sampling). With a sufficient number of iterations, the procedure converges to draws from the joint posterior of all parameters. However, it can be difficult to determine whether convergence has been achieved^{3,29}.

3. Recent Progress

3.1. Incorporating social influence variables

Although HCMs provide additional insight into the decision-making process by considering attitudes and other psychological aspects, they have been developed from individualistic and egocentric perspectives. In other words, the models assume that choice behaviour depends only on individual characteristics (e.g., socio-demographics, personal latent attitudes, etc.) and attributes of alternatives. Recently, travel behaviour researchers have become interested in the effects of social influence on travel choice behaviour^{37,38,39,40}. Since humans are social beings, people are members of social networks and interact with other members of their network. Through interaction with social network members, people acquire information about alternatives and update their expectations of the outcomes of their choices. Therefore, individuals' decisions may be influenced by the choices made by members of their social network.

Several types of social influence may affect individual choice behaviour. As a result of joint activity participation, group decisions, negotiation, etc., which involve exchanging and sharing information with social network members, people may become aware of new alternatives^{41,42,43}. In addition, conformity behaviour may influence choice processes. Individuals may be inclined to mimic the behaviour of others. Mimicking behaviour implies that choices of social network members directly affect an individual's utilities. Hence, "individuals belonging to the same group may tend to behave similarly⁴⁴". This phenomenon has also been referred to as spill-over, conformity and peer effect, social multiplier, cascade, bandwagon effect, imitation, contagion, herd behaviour, and so forth. The impact and intensity of this phenomenon depends on the kind of choice behaviour. When the characteristics of certain choice alternatives are not well-known, or directly experiencing or evaluating the choice options is not easy, people may tend to be more conscious of others' choices. In addition, mimicking behaviour may reflect the tendency to express a similar lifestyle and make similar choices as other members of one's social network.

A few studies in travel behaviour research have incorporated social influence variables into the utility functions, as exogenous explanatory variables. The estimated parameters of these social influence variables indicate how strongly the choices of the reference group influence the decision-making of an individual. Hence, the most important issues in these studies are how to define the reference group and how to measure and represent their choice behaviour. Some studies have applied Brock and Durlauf⁴⁵ seminal choice model, in which the probability of an individual's choice is proportional to the aggregate choice behaviour of the relevant social network. Examples include Dugundji and Walker⁴⁶, Dugundji and Gulyás⁴⁷, Walker et al.⁴⁸ and Dugundji and Gulyás⁴⁹. Various field variables were considered as social influence variables, defined in terms of the average mode share of each alternative in the reference groups. The reference groups were classified according to social and spatial strata, implying that individuals are assumed to be influenced mostly by others who are in similar socio-economic classes and/or live nearby. Fukuda and Morichi⁵⁰ investigated social influence in people's illegal bicycle parking behaviour by employing the aggregate frequencies of illegal bicycle parking at railway stations as the social influence variable.

Páez and Scott⁵¹ and Páez et al.⁵² suggested using the more explicit concept of social networking for defining the reference group. They considered individual-by-individual relationships: one's reference group consists only of one's significant others (i.e., social network members). In particular, Páez et al.⁵² suggested a distant-decay function in terms of social space. They assumed that an individual receives different amount of influence by others depending

on social distance; that is, the social tie strength between individuals. Therefore, their social influence variable represents a socially weighted average choice of social network members.

While these approaches are all based on revealed preference data, Kuwano et al.⁵³ and Rasouli and Timmermans^{54,55} suggested including social influence in stated choice experiments to investigate people's intention to purchase electric cars. Kuwano et al.⁵³ included in their stated choice experiment an attribute depicting the general market share of electric vehicles. Rasouli and Timmermans^{54,55} suggested an experimental approach allowing one to investigate more specific and explicit social network effects by differentiating between different elements of social networks. The social influence variables were defined by market shares of electric cars for various social network types such as family, friend, colleague, and peers social networks. In the stated choice experiment, a respondent indicated his/her intention to buy an electric car for different choice situations varying not only attribute levels of electric cars but also market share levels by the social network types.

By including the social influence variables into the utility function of the discrete choice model with social influence variables, the HCM framework can be elaborated to allow for a mixture of social influences and latent attitudes⁵⁴. The latent variable model part of the elaborated HCM is same as that of the basic HCM, but the utility function of the discrete choice model part (i.e. equation (3)) is changed as follows:

$$U_{in} = \beta_Z(i)X_n^Z(i) + \beta_L(i)X_n^L(i) + \beta_M(i)X_n^M(i) + \beta_S(i)X_n^S(i) + \varepsilon_{in}, \quad (9)$$

where a $(S \times J)$ matrix X_n^S indicates the social influence variables corresponding individual n . The $(J \times S)$ matrix β_S indicates unknown parameters to be estimated for social influence variables, which represents the strength of the social influences. The parameters of the elaborated HCM can be estimated through the same estimation approach used for the basic model. By simultaneously considering the social aspects and the personal latent aspects, the elaborated HCM enables a researcher to investigate not only latent attitudes but social influence in travel choice behaviour.

3.2. Allowing for nonlinear specifications

In the HCM framework, two different types of effects of latent attitudes are modelled. One is the measurement relationship of the latent variable model. The other is the structural relationship of the discrete choice model part. Typically, as described in the basic HCM, linear-in-parameters model specifications have been used for these effects. However, linear relationship is just one of all possible types of effects because a linear function is just a special case of nonlinear functions. Assuming nonlinear relationships thus seems theoretically more appealing as it represents a more general approach to model specification.

In terms of the measurement relationship between latent attitudes and their indicators, Daly et al.⁹ suggested using an ordered choice model by treating the indicators as ordered categorical variables. This approach is an effort to represent and investigate the nonlinear effects of latent attitudes on the attitudinal indicators. However, to date, nonlinear effects of latent attitudes on the utilities have not yet been examined. When the utility functions underlying the discrete choice model are assumed linear in parameters, the model cannot represent any varying marginal utility of the levels of the explanatory variables. Empirical evidence suggests that the marginal disutility for travel tends to increase with travel time and decrease with travel cost^{1,57,58}. Moreover, it may vary with categories of socio-demographic attributes such as income and age^{46,59}. Likewise, different levels of latent attitudes can induce a different marginal utility. For example, the utility of a private car may increase exponentially with an increasing positive latent attitude with respect to privacy. A model that does not take such nonlinear relationships into account might be expected to yield biased policy effects. However, there is no general approach to deal with nonlinear relationships related to latent variables in discrete choice analyses. Therefore, it is necessary to relax the linearity assumption and check for any nonlinearity in the utility functions of HCMs in order to reflect the more general nature of travel choice behaviour.

Several ways to allow nonlinear specification in the utility functions can be considered. First, nonlinear relationships can be approximated by linear-in-parameter specifications. There are two possible approaches as illustrated in Fig. 3. One approach is using piecewise linear functions by segmenting the attribute levels (Fig. 3(a)). By estimating separate piecewise linear functions, we can obtain different parameters for different attribute ranges.

Thus, proper segmentations can approximate a nonlinear utility function. The other approach is to calculate new values for variables by using predetermined nonlinear functions such as logarithmic, exponential or power functions. In other words, the values of variables are exogenously manipulated through a selected functional form, and the manipulated values are used as the explanatory variables. For example, Fig. 3(b) represents that a nonlinear function square function. Before estimating the parameters, the squared value of the explanatory variable is calculated first, and then, by using the squared value, the function can be treated as a linear function.

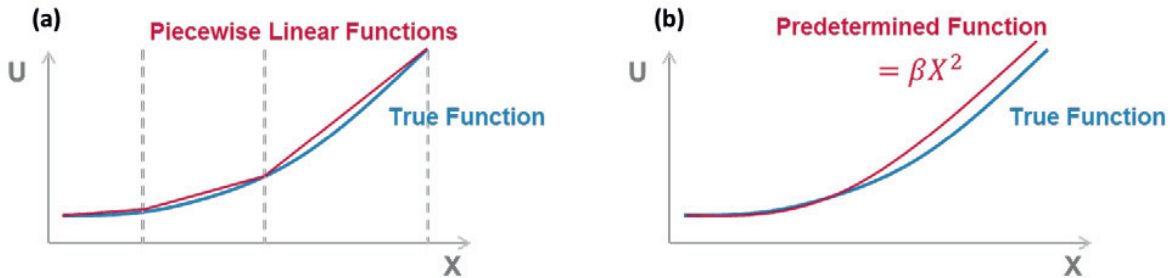


Fig. 3. Approximation of nonlinear function based on linear specification: (a) using piecewise linear functions; (b) using a predetermined function.

These approaches, however, have limitations in that many decisive components such as functional form, the number of piecewise linear functions and range of each segment need to be predetermined before parameters can be estimated. This makes it difficult to find the ideal model to reflect any nonlinear relationships in the data. We can heuristically search for the best combination of these components to find the best model, but this would be an extremely time consuming task. In particular, it is difficult to directly and properly handle the value of a latent variable because the latent variables are treated as random variables and their values are estimated endogenously in the HCM framework. It means that the value depends on the random draws. Moreover, it is varied iteratively according to parameters estimated at each iteration of the estimation procedure. Moreover, there is no diagnostic test to verify which functional form is significantly best. In case of the piecewise linear approach, the utility function is still linear-in-parameters within a selected range and, in general, needs more parameters to describe nonlinear relationships compared to the nonlinear utility function expressed by logarithmic, exponential and/or power functions.

A more general way to consider nonlinear effects is employing nonlinear functions and directly estimating their parameters. First of all, we can easily consider a power function for representing various nonlinear effects of latent variables (e.g., increasing/decreasing marginal effects by increasing/decreasing the value of latent variables). When considering only the latent attitudes and taking a power function for them, the utility function (i.e. equation (3)) of the basic HCM is changed as follows:

$$U_{in} = \beta_Z(i)X_n^Z(i) + \beta_L(i) \begin{bmatrix} X_{in}^1 \lambda_i^1 \\ X_{in}^2 \lambda_i^2 \\ \vdots \\ X_{in}^L \lambda_i^L \end{bmatrix} + \beta_M(i)X_n^M(i) + \varepsilon_{in}, \quad (10)$$

where X_{in}^l indicates l th component of $X_n^L(i)$, and λ_i^l is a transformation parameter of l th latent variable corresponding alternative i . However, there is a mathematical problem in the estimation of the parameters because the function becomes discontinuous at $\lambda_i^l = 0$. In order to overcome this limitation, two transformation rules can be suggested, which are Box-Tukey⁶⁰ and Box-Cox⁶¹ transformations. According to Box-Tukey transformations, the power function can be transformed as follows:

$$X_{in}^l \lambda_i^l = \begin{cases} \frac{(X_{in}^l + \mu_i^l)^{\lambda_i^l} - 1}{\lambda_i^l}, & \lambda_i^l \neq 0, \quad X_{in}^l + \mu_i^l > 0 \\ \ln(X_{in}^l + \mu_i^l), & \lambda_i^l = 0 \end{cases} \quad (11)$$

where μ_i^l is a location parameter corresponding to the l th latent variable. The Box-Cox transformation is obtained as a special case by setting $\mu_i^l = 0$:

$$X_{in}^l \lambda_i^l = \begin{cases} \frac{X_{in}^l \lambda_i^l - 1}{\lambda_i^l}, & \lambda_i^l \neq 0, \quad X_{in}^l > 0 \\ \ln X_{in}^l, & \lambda_i^l = 0 \end{cases} \quad (12)$$

There have been early applications of these transformations for modeling travel behavior. For example, Gaudry and Wills⁶² used Box-Cox transformations to estimate diverse travel demand functions, Gaudry⁶³ explored the properties of inverse transformations, applied the logit and the dogit mode choice models, while Koppelman⁵⁷ applied these transformations in developing non-linear utility functions in mode choice models.

These transformation rules provide several advantages to modeling nonlinear relationships. First, the transformed equations can eliminate the collinearity between the linear parameter β_i^l and the transformation parameter λ_i^l , which exists when directly estimating the power function (i.e. equation 10)^{57,61}. Secondly, the functional form for each variable can be decided endogenously through the estimation result. According to the transformation parameter λ_i^l estimated, the function can approximate linear (i.e. $\lambda_i^l = 1$), logarithmic (i.e. $\lambda_i^l = 0$) and power functions. Finally, this transformation allows carrying out a statistical diagnostic test for functional form. Based on the estimated λ_i^l and its standard error, we can statistically infer whether λ_i^l is significantly different from 0, 1 or both^{58,64}.

There is a constraint in terms of the scales of latent variables. Regarding the Box-Cox transformation, every latent variable corresponding every individual must have only positive values because of the logarithmic component of the utility specification. The scale of latent variable depends on both the scale of relevant indicator variables and the random distribution of the latent variable. When considering the basic HCM, even if the observed scales of indicators are always positive, the probability to take a negative value for a latent variable of an arbitrary individual is not always zero because the normal distribution has infinite ranges in both positive and negative directions. This probability makes it impossible to apply the transformation to latent variables because the entire space of their density functions must be considered in order to evaluate the likelihood function by integrating over the space, as formulated in equation (5). In the case of the Box-Tukey transformation, this kind of problem would be also occurred. The inclusion of the location parameter may be able to reduce the probability to take a negative value for the inclusive term (i.e. $X_{in}^l + \mu_i^l$), but it cannot guarantee that the inclusive term is always positive for the entire space of latent variable.

Therefore, it is necessary to develop a method to deal with this limitation by including an explicit mechanism in the latent variable model of the HCM framework. One possible solution is relevant to assumption of the probability distribution for a random latent variable. If a probability distribution which takes only positive range for their space is considered as the distribution of random disturbance term of a latent variable, it can be guaranteed that the latent variable has only positive value for its entire space. There are several such probability distributions: the lognormal distribution, the Rayleigh distribution, and the truncated normal distribution, etc. In particular, the lognormal distribution, which is a probability distribution whose logarithm has a normal distribution, have been frequently utilized for specifying the distribution of random parameters in the mixed logit model when the parameter is known to have the same sign for every individual, such as cost and travel time^{13,65}. This approach may enable us to use the power transformations to investigate the nonlinear effects of latent attitudes in the HCM framework.

4. Conclusions

The present paper introduces the underlying principles and the recent progresses of HCMs. By incorporating different types of models into discrete choice analysis, the HCM provides a powerful framework to account for heterogeneity across decision makers, due to different latent attitudes. Furthermore, the recent progresses developed by the authors contribute to elaborate and generalize the HCM framework in order to represent a more behaviourally realistic choice behaviour. By including the social influence variables into the utility function of discrete choice model, the HCM framework can take into account peoples' interdependent traits stemming from peoples' sociality. By allowing nonlinear specifications for the utility functions, the HCM framework can be utilized to investigate nonlinear effects of latent variables. In addition, by applying the power transformations which contain a transformation parameter to the utility function, the HCM framework allows approximating various types of nonlinear functions (including linear function) and conducting the statistical diagnostic test of the function form. Therefore it becomes a more generalized approach because the basic HCM is a limiting case of the HCM based on the nonlinear specification.

Although our recent contributions provided an advanced and generalized modeling approach, several directions to further elaborate the modeling approach remain. First, the present study doesn't address a nonlinear relationship between latent variables and their explanatory variables in the HCM framework. By allowing the nonlinear specification for all components of the HCM, a more generalized HCM framework needs to be developed in a future study. Second, HCM based on RRM (or hybrid RRM-RUM) can be further enhanced with including social influence variables and allowing for nonlinear specifications in the regret function. These considerations in the HCM framework will be addressed in a future study.

References

1. Ben-Akiva M, Lerman SR. *Discrete Choice Analysis: Theory and Application to Travel Demand*. Cambridge: MIT Press; 1985.
2. Van Lierop W. *Spatial Interaction Modelling and Residential Choice Analysis*. Gower Publishing Co.; 1986.
3. Ben-Akiva M, McFadden D, Train K, Walker J, Bhat C, Bierlaire M, Bolduc D, Boersch-supan A, Brownstone D, Bunch DS, Daly A, Palma AD, Gopinath D, Karlstrom A, Munizaga MA. Hybrid choice models: Progress and challenges. *Mark Lett* 2002;**13**:163-175.
4. Bolduc D, Ben-Akiva M, Walker J, Michaud A. Hybrid choice models with logit kernel: applicability to large scale models. In: Lee-Gosselin M, Doherty S, editors. *Integrated Land-Use and Transportation Models: Behavioral Foundations*. Oxford: Elsevier; 2005. p. 275-302.
5. Ben-Akiva M, McFadden D, Gärling T, Gopinath D, Walker J, Bolduc D, Boersch-Supan A, Delquié P, Larichev O, Morikawa T, Polydoropoulou A, Rao V. Extended framework for modelling choice behaviour. *Mark Lett* 1999;**10**:187-203.
6. Koppelman F, Hauser J. Destination choice for non-grocery-shopping trips. *Transport Res Rec* 1978;**673**:157-165.
7. Morikawa T. *Incorporating Stated Preference Data in Travel Demand Analysis*. Ph.D. thesis, Massachusetts Institute of Technology; 1989.
8. Elrod T, Keane MP. A factor-analytic probit model for representing the market structure in panel data. *J Market Res* 1995;**32**:1-16.
9. Daly A, Hess S, Patruni B, Potoglou D, Rohr C. Using ordered attitudinal indicators in a latent variable choice model: A study of the impact of security on rail travel behaviour. *Transportation* 2012;**39**:267-297.
10. Golob TF. Structural equation modelling for travel behaviour research. *Transport Res B-Meth* 2003;**37**:1-25.
11. Sakano R, Benjamin JM. A structural equations analysis of revealed and stated travel mode and active choices. *Transportmetrica* 2008;**4**:97-115.
12. Muthén B. A general structural equation model with dichotomous, ordered categorical and continuous latent variable indicators. *Psychometrika* 1984;**49**:115-132.
13. Walker J, Ben-Akiva M. Generalized random utility model. *Math Soc Sci* 2002;**43**:303-343.
14. Dekker T, Hess S, Arentze T, Chorus CG. Incorporating needs-satisfaction and regret-minimization in a discrete choice model of leisure activities. *Proceedings of the 93th Annual Meeting of the Transportation Research Board*, Washington, DC (CD-ROM); 2014.
15. Chorus CG, Arentze TA, Timmermans HJP. A random regret-minimization model of travel choice. *Transport Res B-Meth* 2008;**42**:1-18.
16. Chorus CG, Rose JM, Hensher DA. Regret minimization or utility maximization: it depends on the attribute. *Environ Plann B* 2013;**40**:154-169.
17. Temme D, Paulssen M, Dannewald T. Incorporating latent variables into discrete choice models – A simultaneous estimation approach using SEM software. *Business Research* 2008;**1**:220-237.
18. Kim, J, Bae YK, Chung JH. Effects of personal proenvironmental attitudes on mode choice behavior: New ecofriendly water transit system in Seoul, South Korea. *Transport Res Rec* 2012;**2274**:175-183.
19. Atasoy D, Glerum A, Bierlaire M. Attitudes towards mode choice in Switzerland. *disP – The Planning Review* 2013;**49**:101-117.
20. Kamargianni M, Ben-Akiva M, Polydoropoulou A. Incorporating social interaction into hybrid choice models. *Proceedings of the 93th Annual Meeting of the Transportation Research Board*, Washington, DC (CD-ROM); 2014.

21. Daziano RA, Bolduc D. Incorporating pro-environmental preferences towards green automobile technologies through a bayesian hybrid choice model. *Transportmetrica A* 2013;**9**:74-106.
22. Glerum A, Stankovikj L, Thémans M, Bierlaire M. Forecasting the demand for electric vehicle: accounting for attitudes and perceptions. *Transport Sci* 2013;Articles in advance:1-17.
23. Jensen AF, Cherchi E, Mabit SL. On the stability of preferences and attitudes before and after experiencing an electric vehicle. *Transport Res D-Tr E* 2013;**25**:24-32.
24. Mabit SL, Cherchi E, Jensen AF, Jordal-Jørgensen J. Hybrid choice modeling allowing for reference-dependent preferences: estimation and validation for the case of alternative-fuel vehicles. *Proceedings of the 93th Annual Meeting of the Transportation Research Board*, Washington, DC (CD-ROM); 2014.
25. Soto JJ, Cantillo V, Arellana J. Hybrid choice modeling of alternative fuelled vehicles in Colombian cities including second order structural equations. *Proceedings of the 93th Annual Meeting of the Transportation Research Board*, Washington, DC (CD-ROM); 2014.
26. Prato CG, Bekhor S, Pronello C. Latent variables and route choice behavior. *Transportation* 2012;**39**:299-319.
27. Silva JA, Sottile E, Cherchi E. The effects of land use patterns on tour type choice: the application of a hybrid choice model. *Proceedings of the 93th Annual Meeting of the Transportation Research Board*, Washington, DC (CD-ROM); 2014.
28. McFadden D, Train KE. Mixed MNL models or discrete response. *J App Econom* 2000;**15**:447-470.
29. Train KE. *Discrete Choice Methods with Simulation*. 2nd ed. New York: Cambridge University Press; 2009.
30. Chorus CG. A new model of random regret minimization. *Eur J Transport and Infrastructure* 2010;**10**:181-196.
31. Chorus CG. Random regret minimization: an overview of model properties and empirical evidence. *Transport Rev* 2012;**32**:75-92.
32. Chorus CG, de Jong GC. Modeling experienced accessibility for utility-maximizers and regret-minimizers. *J Transp Geogr* 2011;**19**:1155-1162.
33. Hensher DA, Greene WH, Chorus CG. Random regret minimization or random utility maximization: an exploratory analysis in the context of automobile fuel choice. *J Adv Transport* 2013;**47**:667-678.
34. Chorus CG, Bierlaire M. An empirical comparison of travel choice models that capture preferences for compromise alternatives. *Transportation* 2013;**40**:549-562.
35. Rasouli S, Timmermans HJP. Applications of theories and models of choice and decision-making under conditions of uncertainty in travel behaviour research. *Travel Behavior and Society* 2014;**1**: to appear.
36. Rasouli S. Timmermans HJP. Specification of regret-based models of choice behavior: Formal analyses and experimental design based evidence. *Transportation* 2014;Accepted for publication.
37. Arentze T, Timmermans HJP. Social networks, social interactions, and activity-travel behavior: a framework for microsimulation. *Environ Plann B* 2008;**35**:1012-1027.
38. Axhausen KW. Social networks, mobility biographies, and travel: survey challenges. *Environ Plann B* 2008;**35**:981-996.
39. Dugundji ER, Páez A, Arentze TA, Walker JL. Transportation and social interactions. *Transport Res A-Pol* 2011;**45**:239-247.
40. Ben-Akiva M, de Palma A, McFadden D, Abou-Zeid M, Chiappori P, de Lapparent M, Durlauf SN, Fosgerau M, Fukuda D, Hess S, Manski C, Pakes A, Picard N, Walker J. Process and context in choice models. *Mark Lett* 2012;**23**:439-456.
41. Han Q, Arentze TA, Timmermans HJP, Janssens D, Wets G. Modeling context-sensitive dynamic activity-travel behavior under conditions of uncertainty incorporating reinforcement learning, habit formation, and behavioral and cognitive adaptation strategies. *Proceedings of the 87th Annual Meeting of the Transportation Research Board*, Washington, DC (CD-ROM); 2008.
42. Han Q, Arentze TA, Timmermans HJP, Janssens D, Wets G. The effects of social networks on choice set dynamics: Results of numerical simulations using an agent-based approach. *Transport Res A-Pol* 2011;**45**:310-322.
43. Ettema D, Arentze TA, Timmermans HJP. Social influences on household location, mobility and activity choice in integrated micro-simulation models. *Transport Res A-Pol* 2011;**45**:283-295.
44. Manski C. *Identification Problems in the Social Sciences*. Harvard University Press; 1999.
45. Brock WA, Durlauf SN. Discrete Choice with Social Interactions. *Rev Econ Stud* 2001;**68**:235-260.
46. Dugundji ER, Walker J. Discrete choice with social and spatial network interdependencies: An empirical example using mixed generalized extreme value models with field and panel effects. *Transport Res Rec* 2005;**1921**:70-78.
47. Dugundji ER, Gulyás L. Sociodynamic discrete choice on networks in space: Impacts of agent heterogeneity on emergent outcomes. *Environ Plann B* 2008;**35**:1028-1054.
48. Walker J, Ehlers E, Banerjee I, Dugundji ER. Correcting for endogeneity in behavioral choice models with social influence variables. *Transport Res A-Pol* 2011;**45**:362-374.
49. Dugundji ER, Gulyás L. Sociodynamic discrete choice on networks in space: The role of utility parameters and connectivity in emergent outcomes. *Proceedings of the 92th Annual Meeting of the Transportation Research Board*, Washington, DC (CD-ROM); 2013.
50. Fukuda D, Morichi S. Incorporating aggregate behavior in an individual's discrete choice: An application to analyzing illegal bicycle parking behavior. *Transport Res A-Pol* 2007;**41**:313-325.
51. Páez A, Scott DM. Social influence on travel behavior: A simulation example of the decision to telecommute. *Environ Plann A* 2007;**39**:647-665.
52. Páez A, Scott DM, Volz E. A discrete-choice approach to modeling social influence on individual decision making. *Environ Plann B* 2008;**35**:1055-1069.
53. Kuwano M, Tsukui M, Matsubara T. Analysis on promoting factors of electric vehicles with social conformity. *Presented at the 13th International Conference on Travel Behaviour Research*, Toronto, Canada, 15-20, July 2012.
54. Rasouli S, Timmermans HJP. The effect of social adoption on the intention to purchase electric cars: A stated choice approach, *Transport Res Rec* 2013a;Accepted for publication.

55. Rasouli S, Timmermans HJP. Influence of social networks on latent choice of electric cars: A mixed logit specification using experimental design data. *Networks and Spatial Economics* 2013b;In press.
56. Kim J, Rasouli S, Timmermans HJP. Elaborating hybrid choice models allowing for mixture of social influences and latent attitudes: application to intended purchase of electric cars. *Transport Res A-Pol* 2014;Submitted.
57. Koppelman FS. Non-linear utility functions in models of travel choice behavior. *Transportation* 1981;**10**:127-146.
58. Stathopoulos A, Hess S. Revisiting reference point formation, gains-losses asymmetry and non-linear sensitivities with an emphasis on attribute specific treatment. *Transport Res A-Pol* 2012;**46**:1673-1689.
59. Train KE, McFadden D. The goods/leisure trade off and disaggregate work trip mode choice models. *Transport Res* 1978;**12**:349-353.
60. Tukey JW. On the comparative anatomy of transformations. *Ann Math Stat* 1957;**28**:602-632.
61. Box GEP, Cox DR. An analysis of transformation. *J Roy Stat Soc B Met* 1964;**26**:211-252.
62. Gaudry MJJ, Wills MJ. Estimating the functional form of travel demand models. *Transport Res* 1978;**12**:257-289.
63. Gaudry MJJ. The inverse power transformation logit and dogit mode choice models. *Transport Res B-Meth* 1981;**15**:97-103.
64. Mandel D, Gaudry M, Rothengatter W. Linear or nonlinear utility function in logit models? The impact on German high-speed rail demand forecasts. *Transport Res B-Meth* 1994;**28**:91-101.
65. Brownstone D, Train KE. Forecasting new product penetration with flexible substitution patterns. *J Econometrics* 1999;**89**:109-129.